Reg. No. [ ]

Question Paper Code : 80608

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016

Third Semester

Civil Engineering

MA 6351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches except Environmental Engineering, Textile Chemistry, Textile Technology, Fashion Technology and Pharmaceutical Technology)

(Regulations 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the PDE of all spheres whose centers lie on the x-axis.
2. Find the complete integral of $\frac{z}{pq} = \frac{x}{q} + \frac{y}{p} + \sqrt{pq}$.
3. State the Dirichlet's conditions for a function $f(x)$ to be expanded as a Fourier series.
4. Expand $f(x) = 1$, in $(0,\pi)$ as a half-range sine series.
5. State the assumptions in deriving one-dimensional wave equation.
6. State the three possible solutions of the one-dimensional heat flow (unsteady state) equation.
7. State change of scale property on Fourier transforms.
8. Find the infinite Fourier sine transform of $f(x) = \frac{1}{x}$
9. State convolution theorem on Z-transform.
10. Find $Z\left[\frac{1}{n(n+1)}\right]$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the partial differential equations of all planes which are at a constant distance 'k' units from the origin. (8)

(ii) Solve the Lagrange's equation $x(z^2-y^2)p + y(x^2-z^2)q = z(y^2-x^2)$. (8)

Or

(b) (i) Form the PDE by eliminating the arbitrary functions 'f' and 'φ' from the relation $z = xf\left(\frac{y}{x}\right) + y\varphi(x)$. (8)

(ii) Solve $(D^2 + DD' - 6D'^2)z = y\cos x$. (8)

12. (a) (i) Expand $f(x) = x^2$ as a Fourier series in the interval $(-\pi,\pi)$ and hence deduce that $1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \ldots = \frac{\pi^4}{90}$. (8)

(ii) Obtain the constant term and the coefficient of the first sine and cosine terms in the Fourier expansion of y as given in the following table: (8)

x	0	1	2	3	4	5
y	9	18	24	28	26	20

Or

(b) (i) Expand $f(x) = e^{-ax}, -\pi < x < \pi$ as a complex form Fourier series. (8)

(ii) Expand $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \end{cases}$ as a series of cosines in the interval $(0,2)$. (8)

13. (a) A tightly stretched string of length 'l' with fixed end points is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $y_t(x,0) = v_0 \sin\left(\frac{3\pi x}{l}\right)\cos\left(\frac{\pi x}{l}\right)$, where $0 < x < l$. Find the displacement of the string at a point, at a distance x from one end at any instant 't'. (16)

Or

(b) A square plate is bounded by the lines $x = 0, x = 20, y = 0, y = 20$. Its faces are insulated. The temperature along the upper horizontal edge is given by $u(x,20) = x(20 - x), 0 < x < 20$, while the other three edges are kept at 0°C. Find the steady state temperature distribution $u(x,y)$ in the plate. (16)

14. (a) (i) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ and hence deduce that $\int_0^{\infty} \left[\frac{\sin t}{t}\right]^4 dt = \frac{\pi}{3}$. (8)

(ii) Find the infinite Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$ hence deduce the infinite Fourier sine transform of $\frac{1}{x}$. (8)

Or

(b) (i) Find the infinite Fourier transform of $e^{-a^2x^2}$ hence deduce the infinite Fourier transform of $e^{-x^2/2}$. (8)

(ii) Solve the integral equation $\int_0^{\infty} f(x)\cos\lambda x \, dx = e^{-\lambda}$, where $\lambda > 0$. (8)

15. (a) (i) Find

(1) $Z[n^3]$ (2) $Z[e^{-t}t^2]$. (4+4)

(ii) Evaluate $Z^{-1}\left[\frac{9z^3}{(3z-1)^2(z-2)}\right]$, using calculus of residues. (8)

Or

(b) (i) Using convolution theorem, evaluate $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$. (8)

(ii) Using Z-transform, solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ given that $y_0 = y_1 = 0$. (8)